Question Paper

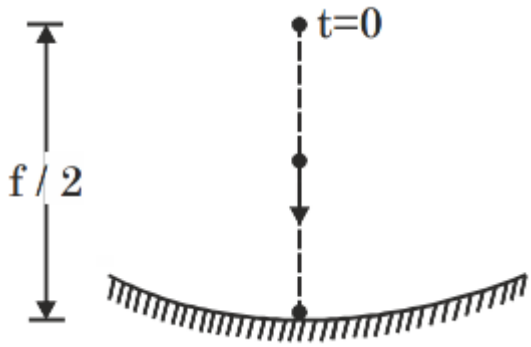
Physics Single Correct (Maximum Marks: 15)

Question No. 1

Only One Option Correct Type

Each question has multiple options out of which ONLY ONE is correct.

A particle is dropped along the axis from a height $f/2$ on a concave mirror of focal length f as shown in figure. The acceleration due to gravity is g. The maximum speed of image is $\frac{3}{4}\sqrt{nfg}$. Find n



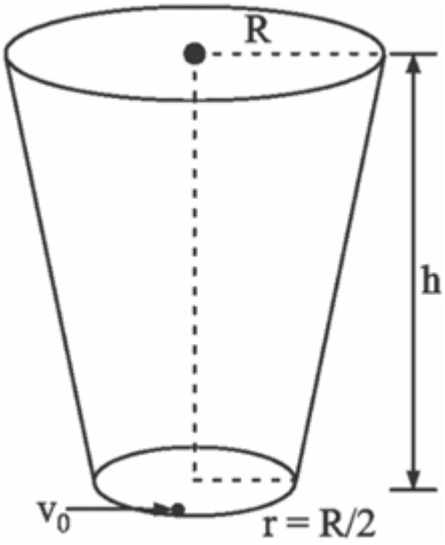
A. 1
B. 2
C. 3
D. 4

Question No. 2

Only One Option Correct Type

Each question has multiple options out of which ONLY ONE is correct.

A small particle of mass m lies on the bottom surface of smooth conical glass surface, as shown in the figure. The particle touches the circumference. The minimum velocity v_0 (along the circumference) such that particle reaches at top of surface is :



A. $\sqrt{\frac{8}{3}gh}$

B. $2\sqrt{gh}$

C. $\sqrt{\frac{2}{3}gh}$

D. $\sqrt{gh}$

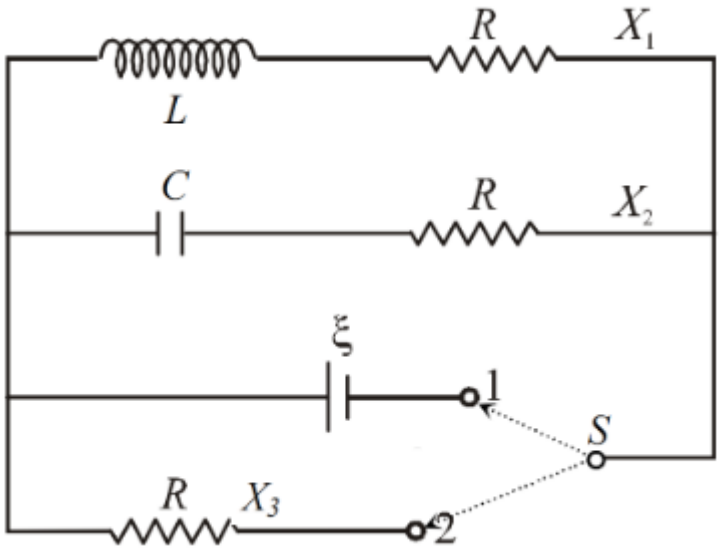
Question No. 3

Only One Option Correct Type

Each question has multiple options out of which ONLY ONE is correct.

Figure shows a circuit containing three resistor X_1, X_2 &X_3 having resistance R each, an inductor, capacitor and an emf source having inductance 'L', capacitance C \& emf ξ respectively (Given $R = \sqrt{\frac{L}{C}}$). The switch is first connected to position-1. When charge on capacitor becomes half of it maximum possible value then switch S is connected to position-2. Current in resistance X_3 just after shifting the switch from position-1 to position-2, is

(given L = 5mH, C = 2μF, R = 10Ω, ξ = 5 V)



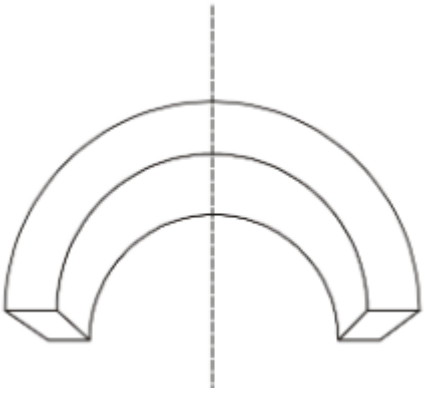
A. 1
B. 1.5
C. 2.5
D. None of these

Question No. 4

Only One Option Correct Type

Each question has multiple options out of which ONLY ONE is correct.

A conducting rod of square cross section is bent in a semi circular shape of inner radius ' r ' as shown in figure. Length of side of cross section is equal to ' a '. Resistivity of material is ' ρ '. The resistance of rod across its end will be



A. $\frac{\pi\rho}{a\ln(1+a/r)}$

B. $\frac{2\pi\rho}{a\ln(1+a/r)}$

C. $\frac{\pi\rho(r+a)}{a^2}$

D. $\frac{2\pi r\rho}{a^2}$

Question No. 5

Only One Option Correct Type

Each question has multiple options out of which ONLY ONE is correct.

A thermally insulated piece of metal is heated under atmospheric pressure by an electric current so that it receives electric energy at a constant power P. This leads to an increase of the absolute temperature T of the metal with time t as given by the equation $T(t) = T_0[1 + a(t - t_0)]^{1/4}$. Here a, t_0 and T_0 are constants. The heat capacity of the metal is $C_P(T) = \frac{4P}{aT_0^4} T^n$ (which is temperature dependent in the temperature range of the experiment), then find the value of 'n'.

A. 9

B. 3

C. 7

D. -1

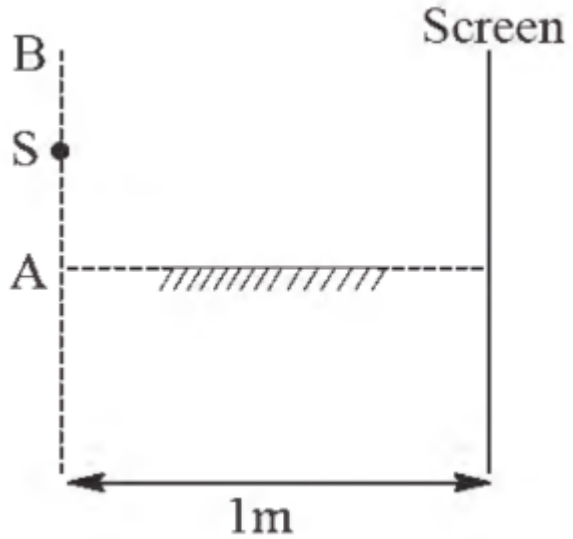
Physics Multiple Correct (Maximum Marks: 28)

Question No. 1

One or More Options Correct Type

The question has multiple options out of which ONE or MORE is/are correct.

In an interference experiment as shown in the figure, the source plane and screen are separated by a distance 1 m. At a certain position of source, fringe width is 1/4 mm and by moving the source away from mirror along the line AB by 0.6 mm, the fringe width changed to 1/6 mm :



A. Wavelength of light used is 5000Å
B. Wavelength of light used is 6000Å
C. Initial distance of source from A is 1.2 mm
D. Initial distance of source from A is 0.6 mm

Question No. 2

One or More Options Correct Type

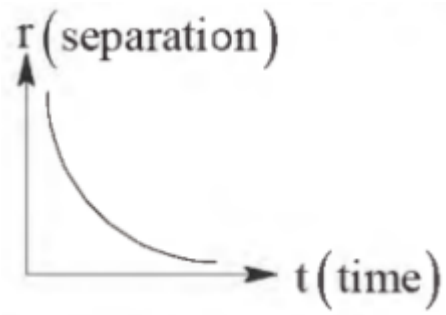
The question has multiple options out of which ONE or MORE is/are correct.

A radioactive point source has a decay constant λ. When this source moves towards small area counter kept at large distance from source, then counter records count/second which turns out to be constant:

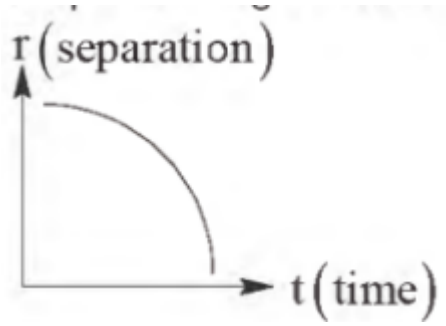
A. When separation between source and counter becomes half of initial value, number of nuclei left undecayed will be $\frac{3}{4}$ th of initial value

B. When separation between source and counter becomes half of initial value, number of nuclei left undecayed will be $\frac{1}{4}$ th of initial value

C. Graph showing variation of separation between source and counter with time is as shown



(D) Graph showing variation of separation between source and counter with time is as shown

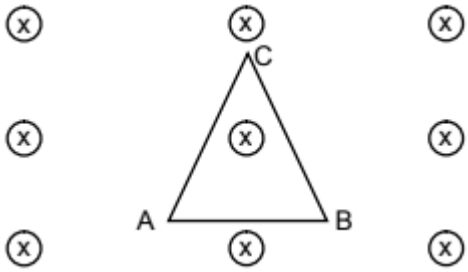


Question No. 3

One or More Options Correct Type

The question has multiple options out of which ONE or MORE is/are correct.

ABC is an equilateral triangle, of side length ℓ_0 and resistance R_0 at temperature $0^\circ C$, kept in uniform magnetic field B_0 as shown.



Temperature coefficient of linear expansion of wire is α_t and temperature coefficient of resistance of wire is α_R. Now the temperature of the wire is slowly changed according to the equation $T = T_0 \sin \omega t$, where T_0 is in $^\circ C$. Given that $\alpha_R = \alpha_\ell = \alpha$ and $\alpha_R T_0 \ll 1 \& \alpha_\ell T_0 \ll 1$). Choose the correct option(s):

A. Amount of charge flown through the wire in the first quarter cycle of temperature starting from $t = 0$ is $\left(\frac{\sqrt{3} B_0 \alpha \ell_0^2}{2R_0}\right) T_0$

B. Amount of charge flown through the wire in the first quarter cycle of temperature starting from $t = 0$ is $\left(\frac{\sqrt{3} B_0 \alpha \ell_0^2}{4R_0}\right) T_0$

C. Rate of heat production from wire at any time 't' is $\frac{3}{4} \frac{B_0^2 \alpha^2 \ell_0^4 \omega^2 T_0^2}{R_0} e^{3\alpha T} \cos^2 \omega t$

D. Rate of heat production from wire at any time 't' is $\frac{3}{2} \frac{B_0^2 \alpha^2 \ell_0^4 \omega^2 T_0^2}{R_0} e^{3\alpha T} \cos^2 \omega t$

Question No. 4

One or More Options Correct Type

The question has multiple options out of which ONE or MORE is/are correct.

A charged particle is projected in magnetic field $\vec{B} = 10\hat{k}$ from origin in $x - y$ plane. The particle moves in a circle and just touches a line $y = 5$ m at $x = 5\sqrt{3}$ m . Then (mass of particle $= 5 \times 10^{-5}$ kg , charge $= 1 \mu c$) -

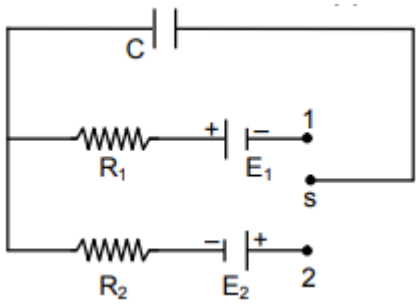
A. The particle is projected at an angle 60° with x-axis
B. The radius of circle is 10 m
C. speed of particle is 2 m/s
D. workdone by magnetic force on the particle is zero.

Question No. 5

One or More Options Correct Type

The question has multiple options out of which ONE or MORE is/are correct.

An uncharged capacitor C, resistances $R_1 = R$ and $R_2 = 2R$, cells of emf $E_1 = E$ and $E_2 = E$ are connected as shown in the figure. First switch S is connected to terminal 1 for a long time. H_1 is heat energy lost through the resistor R_1 up to the steady state and U_1 is energy stored in the capacitor at steady state. Now the switch is connected to terminal 2 for an another long duration. H_2 is heat energy lost through the resistor R_2 up to the steady state and U_2 is energy stored in the capacitor at steady state. Choose the correct statement(s)



A. $\frac{U_1}{U_2} = 1$

B. $\frac{H_1}{H_2} = \frac{1}{4}$

C. $H_2 = 2H_1$

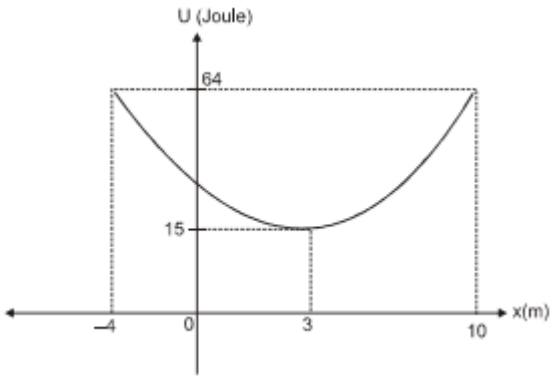
D. $H_2 = CE^2$

Question No. 6

One or More Options Correct Type

The question has multiple options out of which ONE or MORE is/are correct.

A single conservative force $F(x)$ acts on a particle that moves along the x-axis. The graph of the potential energy with x is given. At $x = 5$ m, the particle has a kinetic energy of 50 J and its potential energy is related to position 'x' as $U = 15 + (x-3)^2$ Joule, where x is in meter. Then :



A. The mechanical energy of system is 69 J.

B. The mechanical energy of system is 19 J.

C. At $x = 3$, the kinetic energy of particle is minimum

D. The maximum value of kinetic energy is $54\ \text{J}$.

Question No. 7

One or More Options Correct Type

The question has multiple options out of which ONE or MORE is/are correct.

When photons of energy 4.25eV strike the surface of a metal A, the ejected photoelectrons have maximum kinetic energy $T_A\ \text{eV}$ with de-Broglie wavelength λ_A. The maximum kinetic energy of photoelectrons liberated from another metal B by photons of energy 4.70eV is $T_B = (T_A - 1.50\text{eV})$. If the de-Broglie wavelength of these photoelectrons is $\lambda_B = 2\lambda_A$, then :

A. the work function of A is 2.25eV
B. the work function of B is 4.20eV
C. $T_A = 2.00\text{eV}$
D. $T_B = 2.75\text{eV}$

Physics Numerical (Maximum Marks: 24)

Question No. 1

Numerical Type

The answer has to be filled into the input box provided below.

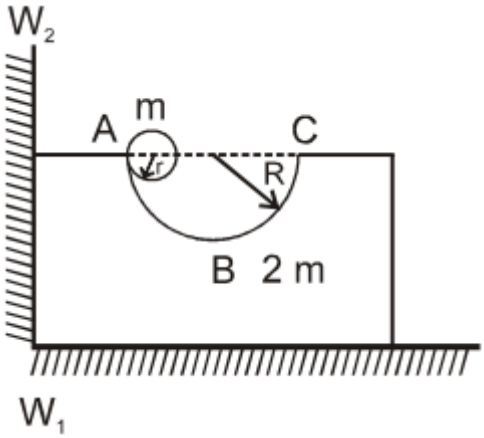
A point moves according to the law $x = at, y = at(1 - \alpha t)$ where a and α positive constants are and t is time. If the moment at which angle between velocity vector and acceleration vector is $\frac{\pi}{4}$ is given by $\frac{A}{\alpha}$ is given by $\frac{A}{\alpha}$. Find the value of A.

Question No. 2

Numerical Type

The answer has to be filled into the input box provided below.

A uniform solid sphere of mass m and radius r is released from a wedge of mass $2m$ as shown. ABC is hemispherical position of radius R. Impulse imparted to the system consisting wedge and sphere by the vertical wall $w_1 w_2$ till the time sphere reaches at the bottom most position of spherical portion for the first time is $m\sqrt{\frac{10g(R-r)}{\delta}}$. Friction between wedge and horizontal surface is absent and between sphere and wedge friction is sufficient to avoid slipping between them. Here δ is an integer. Find δ.



Question No. 3

Numerical Type

The answer has to be filled into the input box provided below.

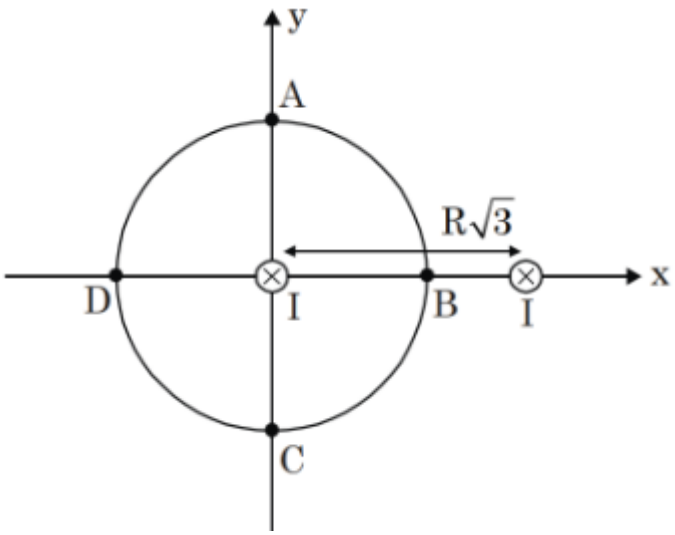
Two rays are incident on a spherical concave mirror of radius $R = 5$ cm parallel to its optical axis at perpendicular distances 3 cm and 4 cm respectively. Determine the value Δx if distance between the points at which these rays intersect the optical axis after being reflected from the mirror is $\frac{\Delta x}{24}$ cm.

Question No. 4

Numerical Type

The answer has to be filled into the input box provided below.

Two current carrying wire of infinite length are placed perpendicular to x − y plane at x = 0 and x = R$\sqrt{3}$ as shown in figure. Current in both the wires is same and inwards to the $x - y$ plane. Find ratio of $\left|\int \vec{B} \cdot \vec{d\ell}\right|$ in segment CDA to segment ABC will be :



Question No. 5

Numerical Type

The answer has to be filled into the input box provided below.

In a searle's experiment, the diameter of the wire as measured by a screw gauge is 0.050 cm. Pitch of screw gauge is 2 mm and number of divisions on circular scale are 200. The length, measured by a scale of least count 0.1 cm is 100 cm. When a weight of 50 N is suspended from the wire elongation is measured to be 0.125 cm by a micrometer of least count 0.001 cm. Find the maximum percentage error is the measurement of young's modulus of the material of the wire from these data.

Question No. 6

Numerical Type

The answer has to be filled into the input box provided below.

If the sound heard by observer, whose equation is given as $y = 8 \sin 10\pi t \cos 200\pi t$ at x = 0 , then number of beat frequency heard by observer is 2k. Then the value of k is

Chemistry Single Correct (Maximum Marks: 15)

Question No. 1

Only One Option Correct Type

Each question has multiple options out of which ONLY ONE is correct.

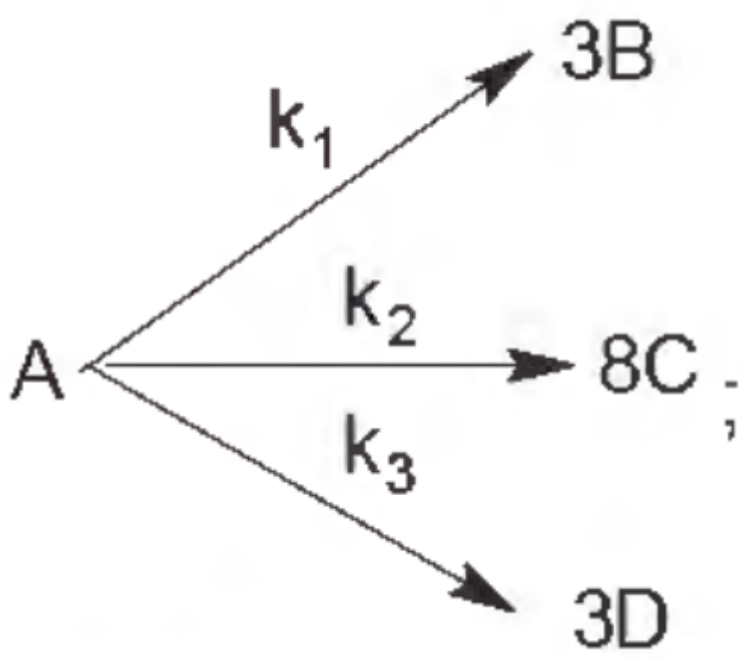
The vapour pressure of an aqueous solution is found to be 750 torr at a temperature TK & the same solution show an elevation in boiling point equal to $1.04\ K$. If TK is the boiling point of pure water, then the atmospheric pressure should be (Given: $K_b(H_2O) = 0.52\ K \cdot kg \cdot mol^{-1}$)

A. 760 torr
B. 777 torr
C. 746 torr
D. 750 torr

Question No. 2

Only One Option Correct Type

Each question has multiple options out of which ONLY ONE is correct.



At time $t = 0$, initial mole of A is 1.

Overall half life of the reaction 15 days. Then calculate the number of mole of C after 45 days if the raetio of $k_1 : k_2 : k_3$ is $4 : 2 : 1$ (assume all reactions follow first order kinetics)

A. 1
B. 2
C. 3
D. 4

Question No. 3

Only One Option Correct Type

Each question has multiple options out of which ONLY ONE is correct.

When H_2S is passed in $Ba(OH)_2$ solution:

A. milkyness is produced due to formation of insoluble salt
B. no change is observed because H_2S does not reaction with $Ba(OH)_2$
C. milkyness is produced due to the formation of $BaSO_3$
D. no change is observed due to the formation of water soluble salt

Question No. 4

Only One Option Correct Type

Each question has multiple options out of which ONLY ONE is correct.

For a complex (d^6 - configuration) having $\Delta_0 = 25000\ cm^{-1}$ and $P = 15000\ cm^{-1}$, the crystal field stabilisation energy is:

A. $30,000\ cm^{-1}$
B. $-60,000\ cm^{-1}$
C. $-30,000\ cm^{-1}$
D. $-60,000\ cm^{-1}$

Question No. 5

Only One Option Correct Type

Each question has multiple options out of which ONLY ONE is correct.

A compound has the empirical formula $C_{10}H_8Fe$. A solution of $0.26\ g$ of the compound in $11.2\ g$ of benzene (C_6H_6) boils at $80.26^\circ C$. The boiling point of benzene is $80.10^\circ C$; the K_b is $2.53^\circ C/$ molal. What is the molecular formula of the compound?

A. $C_{30}H_{24}Fe_3$
B. $C_{10}H_8Fe$
C. C_5H_4Fe
D. $C_{20}H_{16}Fe_2$

Chemistry Multiple Correct (Maximum Marks: 28)

Question No. 1

One or More Options Correct Type

The question has multiple options out of which ONE or MORE is/are correct.

In a solid AB having CsCl type structure, if we reverse the pattern of arrangement of AB i.e. A^+ will be where B^- was and B^- is where A^+ was, then which of the following statement is/are true for new structure thus obtained :

A. C.N of A^+ will be 8
B. C.N of B^- will be 8
C. next nearest neighbours of B^- will be 6

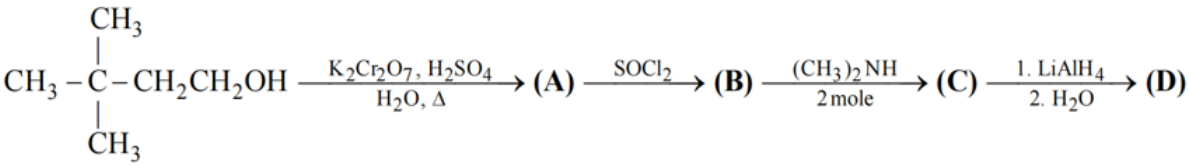
D. formula of compound will be AB_2

Question No. 2

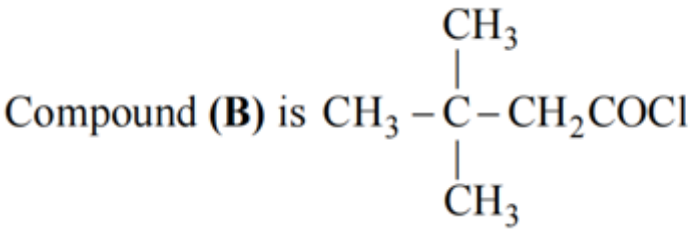
One or More Options Correct Type

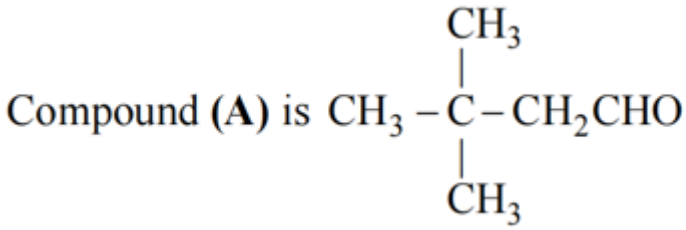
The question has multiple options out of which ONE or MORE is/are correct.

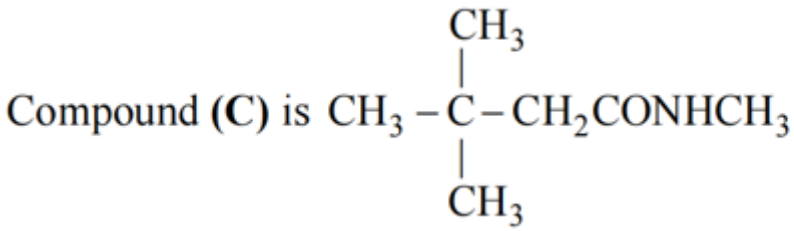
In the given reaction sequence,

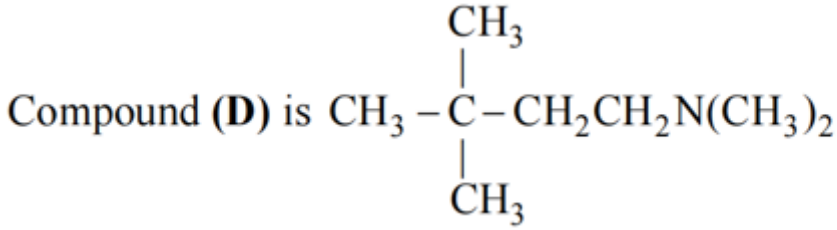


Identify the given compounds :

A. Compound **(B)** is 

B. Compound **(A)** is 

C. Compound **(C)** is 

D. Compound **(D)** is 

Question No. 3

One or More Options Correct Type

The question has multiple options out of which ONE or MORE is/are correct.

Calculate the enthalpy change when infinitely dilute solutions of $CaCl_2$ and Na_2CO_3 are mixed. ΔH_f° for $Ca^{2+}(aq), CO_3^{2-}(aq)$ and $CaCO_3(s)$ are $-129.80, -161.65$ and $-288.45 kcal mol^{-1}$ respectively.

A. 1kcal

B. 3kcal
C. 5kcal
D. None of these

Question No. 4

One or More Options Correct Type

The question has multiple options out of which ONE or MORE is/are correct.

The correct statement(s) about reaction of X_2 molecules of group 17 elements is(are) :

A. Yellow coloured F_2 reacts with water to gives colourless solution and paramagnetic gas
B. Greenish yellow coloured Cl_2 reacts with water to gives colourless acidic solution
C. Reddish brown coloured Br_2 when dissolved in Na_2CO_3 solution form colourless solution
D. Violet coloured iodine when dissolved in water having KI in presence of starch form blue coloured solution

Question No. 5

One or More Options Correct Type

The question has multiple options out of which ONE or MORE is/are correct.

Addition of excess ammonia to a light blue coloured aqueous solution of $CuSO_4 \cdot 5H_2O(X)$ gives a complex (Y). The reaction of (X) with excess of KCN results in the formation of a colourless complex (Z) and a gaseous substance (P). Among the following options, which statement(s) is(are) correct?

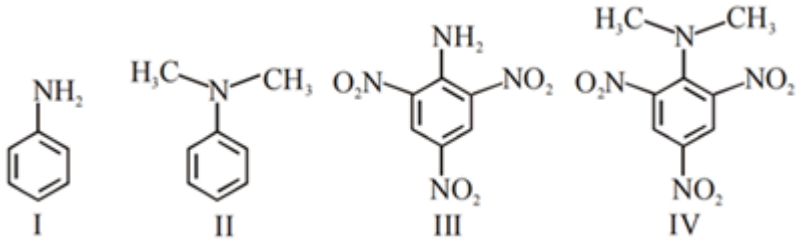
A. Four water molecules are coordinated in $CuSO_4 \cdot 5H_2O$
B. (Y) is a square planar complex
C. The hybridization of the central metal ion in (Z) is sp^3
D. The calculated spin only magnetic moment of complex (Z) and gaseous substance (P) is zero

Question No. 6

One or More Options Correct Type

The question has multiple options out of which ONE or MORE is/are correct.

Consider the following four compounds I, II, III, and IV.



Choose the correct statement(s).

A. The order of basicity is II > I > III > IV.
B. The magnitude of pK_b difference between I and II is more than that between III and IV.
C. Resonance effect is more in III than in IV.
D. Steric effect makes compound IV more basic than III.

Question No. 7

One or More Options Correct Type

The question has multiple options out of which ONE or MORE is/are correct.

Sodium bicarbonate is less soluble in water than potassium bicarbonate, it is due to

A. low molecular weight of $NaHCO_3$ as compared to $KHCO_3$.
B. hydrogen bonding; sodium bicarbonate contains a dimeric anionic structure, in case of potassium bicarbonate anions form an infinite chain.
C. hydrogen bonding; potassium bicarbonate contains a dimeric anionic structure while in sodium bicarbonate anions form an infinite chain.
D. $KHCO_3$ is thermally less stable than $NaHCO_3$.

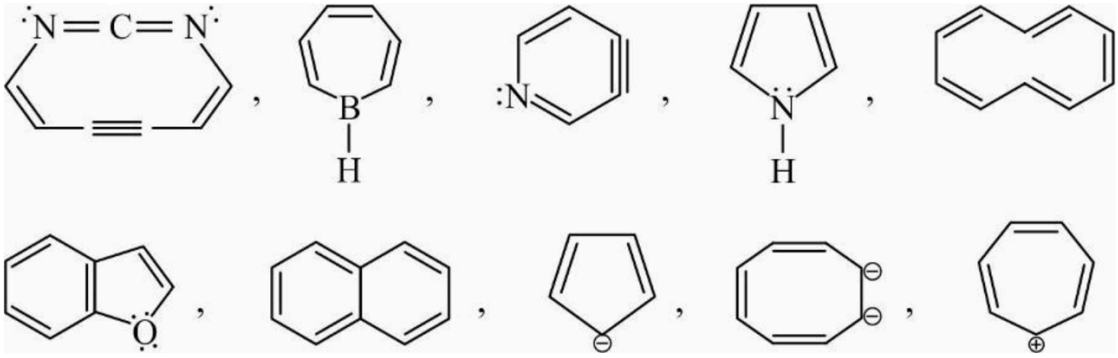
Chemistry Numerical (Maximum Marks: 24)

Question No. 1

Numerical Type

The answer has to be filled into the input box provided below.

Among the following, the number of aromatic compound(s) having only sp^2 hybridized atoms is



Question No. 2

Numerical Type

The answer has to be filled into the input box provided below.

Select the number of ores for which roasting process is applied during metallurgy.

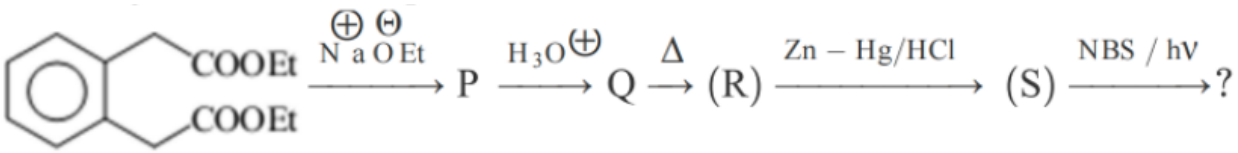
Argentite, Copper pyrites, Galena, Zinc blende, Chalcocite

Question No. 3

Numerical Type

The answer has to be filled into the input box provided below.

For the given reaction sequence :



Number of monohalogenated products is/are -

Question No. 4

Numerical Type

The answer has to be filled into the input box provided below.

Gaseous nitrosyl chloride and nitrogen are taken in a flask sealed and heated to some temperature where the total pressure would have been 1 bar if the following equilibrium had not been established. $2NOCl(g) \overset{K_p}{\rightleftharpoons} 2NO(g) + Cl_2(g)$

But the actual pressure was found to be 1.2 bar due to above equilibrium .Now into the above equilibrium mixture, some Cl_2 gas was introduced at constant V& T so that the total pressure would have been 8.3 bar if no further reaction had occurred but the actual pressure was found to be 8.2 bar. find the value of K_P.

Question No. 5

Numerical Type

The answer has to be filled into the input box provided below.

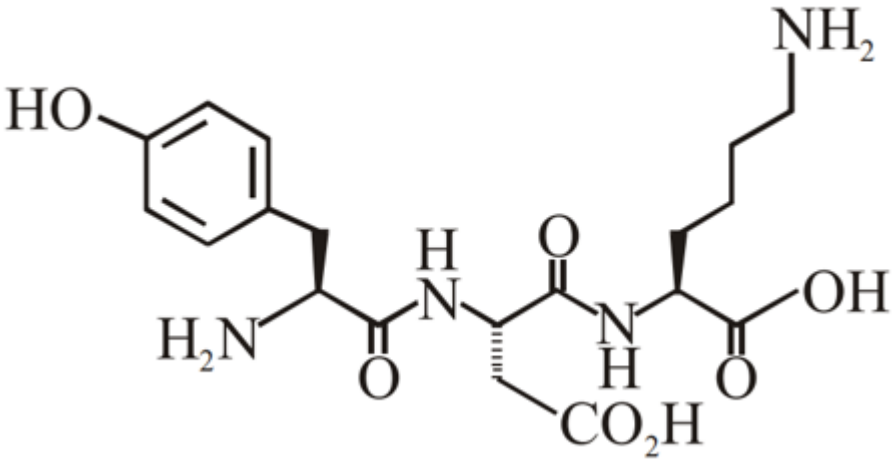
In the molecular structure of $Fe_2(CO)_9$, The difference between the number of triply bonded C ≡ O units and doubly bonded C = O units are/is how many.

Question No. 6

Numerical Type

The answer has to be filled into the input box provided below.

The structure of a peptide is given below.



If the absolute values of the net charge of the peptide at pH = 2, pH = 6 , and pH = 11 are $|z_1|, |z_2|$ and $|z_3|$, respectively, then what is $|z_1| + |z_2| + |z_3|$?

Mathematics Single Correct (Maximum Marks: 15)

Question No. 1

Only One Option Correct Type

Each question has multiple options out of which ONLY ONE is correct.

The term independent of x any y in the expansion of $[(\sqrt{x}+\frac{1}{\sqrt{x}})^2+(\sqrt{y}+\frac{1}{\sqrt{y}})^2+(\sqrt{xy}+\frac{1}{\sqrt{xy}})^2-4]^n$ is

A. $(\sum_{r=0}^{n} {}^nC_r)^2$
B. $\sum_{r=0}^{n} ({}^nC_r)^2$
C. $(\sum_{r=0}^{n} {}^nC_r)^3$
D. $\sum_{r=0}^{n} ({}^nC_r)^3$

Question No. 2

Only One Option Correct Type

Each question has multiple options out of which ONLY ONE is correct.

Consider a curve $|x|+|y|=1$ such that lines $y=mx, y=nx$ make points of intersection in the same quadrant. Let A, B be two such points lying in the same quadrant such that OA, AB , OB are in A.P. If θ is angle between OA and OB then the maximum angle θ is (0 is origin)

A. $\frac{\pi}{6}$
B. $\frac{\pi}{3}$
C. $\frac{\pi}{4}$
D. None of these

Question No. 3

Only One Option Correct Type

Each question has multiple options out of which ONLY ONE is correct.

A line L cuts the lines AB, AC and AD of a parallelogram ABCD at points B_1, C_1 and D_1 respectively. If $\overrightarrow{AB_1}=\lambda_1\overrightarrow{AB}$, $\overrightarrow{AD_1}=\lambda_2\overrightarrow{AD}$ and $\overrightarrow{AC_1}=\lambda_3\overrightarrow{AC}$ then $\frac{1}{\lambda_3}=$

A. $\frac{1}{\lambda_1}+\frac{1}{\lambda_2}$
B. $\frac{1}{\lambda_1}-\frac{1}{\lambda_2}$
C. $-\lambda_1+\lambda_2$
D. $\lambda_1+\lambda_2$

Question No. 4

Only One Option Correct Type

Each question has multiple options out of which ONLY ONE is correct.

Let the circle $(x-1)^2+(y-2)^2=25$ cuts rectangular hyperbola with transverse axis along $y=x$ at 4 points A, B, C and D having coordinates $(x_i, y_i), i=1,2,3,4$ such that origin is centre of hyperbola. If $\ell = x_1+x_2+x_3+x_4,\ m = x_1^2+x_2^2+x_3^2+x_4^2, n = y_1^2+y_2^2+y_3^2+y_4^2$ then $\frac{m+n}{\ell}$ =

A. 100
B. 10
C. 50
D. 20

Question No. 5

Only One Option Correct Type

Each question has multiple options out of which ONLY ONE is correct.

Let $A_1, A_2, A_3, \ldots\ldots A_n$ be vertices of n sided regular polygon inscribed in a circle of radius unity. If P_i be a point lying on the circle of radius ' i ' and concentric with given regular polygon, then

A. $\prod_{i=1}^{n-1} A_iA_{i+1} = n$
B. $\sum_{i=1}^{n} (A_1A_i) = 1$
C. $\sum_{i=1}^{n} (P_1A_i)^2 = 2n$
D. $\sum_{k=1}^{n} \sum_{i=1}^{n} (P_kA_i)^2 = n + \frac{n(n+1)(2n+1)}{6}$

Mathematics Multiple Correct (Maximum Marks: 28)

Question No. 1

One or More Options Correct Type

The question has multiple options out of which ONE or MORE is/are correct.

Two parabola $P_1 : (y-2)^2 = 16(x-1)$ and $P_2 : (y-2)^2 = -20(x-10)$ intersects each other at P and Q, tangent at P to P_1 and P_2 intersects the axis of the parabola at M and N respectively, also tangents at P and Q to P_1 intersects each other at R and tangents at P and Q to P_2 intersects each other at S, then which of the following is/are true?

A. $MN = 18$ and radius of circum circle of ΔMPN is 9
B. If $SR = a$, then radius of circumcircle of ΔSQR is $\frac{a}{\sqrt{2}}$
C. If centre of circumcircle of ΔMPN is (α, β) then $|\alpha| + |\beta| = 7$
D. FP, FN and FM are in H.P., where F is the focus of P_2

Question No. 2

One or More Options Correct Type

The question has multiple options out of which ONE or MORE is/are correct.

The coordinates of a point on the line $\frac{x-1}{2} = \frac{y+1}{-3} = z$ at a distance $4\sqrt{14}$ from the point $(1, -1, 0)$ are

A. $(9, -13, 4)$
B. $(8\sqrt{14} + 1, -12\sqrt{14} - 1, 4\sqrt{14})$
C. $(-7, 11, -4)$
D. $(-8\sqrt{14} + 1, 12\sqrt{14} - 1, -4\sqrt{14})$

Question No. 3

One or More Options Correct Type

The question has multiple options out of which ONE or MORE is/are correct.

Let the end points M and N of the diameter $4x - y = 15$ of the circle $x^2 + y^2 - 6x + 6y - 16 = 0$ are also on the tangents at the end points of the major axis of the ellipse respectively such that MN is also tangent to the same ellipse at point ' P '. If the major axis of the ellipse is along the line $y = x$, then

A. Eccentricity of the ellipse is $\frac{4}{5}$
B. Length of the latus rectum is $\frac{18}{5}$
C. Center of the ellipse is at $(1, 1)$
D. Equation of one of the directrix will be $x + y = \frac{25\sqrt{2}}{4}$

Question No. 4

One or More Options Correct Type

The question has multiple options out of which ONE or MORE is/are correct.

If $\sin(2\cos^{-1}(\cot(2\tan^{-1}x)) = 0$ then possible values of x is/are

A. $1, -1$
B. $-1 + \sqrt{2}, -1 - \sqrt{2}$
C. $1 + \sqrt{2}, 1 - \sqrt{2}$
D. $\sqrt{2}, -\sqrt{2}$

Question No. 5

One or More Options Correct Type

The question has multiple options out of which ONE or MORE is/are correct.

Consider the function $f(x) = \sin^5 x + \cos^5 x - 1,\ x \in [0, \frac{\pi}{2}]$. Which of the following is/are correct?

A. f is monotonic increasing in $(0, \frac{\pi}{4})$.
B. f is monotonic decreasing in $(\frac{\pi}{4}, \frac{\pi}{2})$.

C. $\exists\, c \in (0, \frac{\pi}{4}) f'(c) = 0$

D. The equation $f(x) = 0$ has two roots in $[0, \frac{\pi}{2}]$.

Question No. 6

One or More Options Correct Type

The question has multiple options out of which ONE or MORE is/are correct.

If $\vec{p} = \frac{\vec{b}\times\vec{c}}{[\vec{a}\ \vec{b}\ \vec{c}]}$, $\vec{q} = \frac{\vec{c}\times\vec{a}}{[\vec{a}\ \vec{b}\ \vec{c}]}$, $\vec{r} = \frac{\vec{a}\times\vec{b}}{[\vec{a}\ \vec{b}\ \vec{c}]}$, where $\vec{a}, \vec{b}$ and $\vec{c}$ are any three vectors forming a right-handed system, then which of the following options are correct (where, x is any positive real number)?

A. $[\vec{p}\ \vec{q}\ \vec{r}] = \frac{1}{[\vec{a}\ \vec{b}\ \vec{c}]}$

B. $[\vec{p}\ \vec{q}\ \vec{r}] > 0$

C. $x[\vec{a}\ \vec{b}\ \vec{c}] + \frac{[\vec{p}\ \vec{q}\ \vec{r}]}{x} \geq 2, \forall\, x > 0$

D. $x^4[\vec{a}\ \vec{b}\ \vec{c}]^2 + \frac{[\vec{p}\ \vec{q}\ \vec{r}]}{x^2} \geq (\frac{3}{2^{2/3}})$

Question No. 7

One or More Options Correct Type

The question has multiple options out of which ONE or MORE is/are correct.

Let S be the set of all non-zero real numbers α such that the quadratic equation $\alpha x^2 - x + \alpha = 0$ has two distinct real roots x_1 & x_2 satisfying the inequality $|x_1 - x_2| < 1$. Which of the following intervals is(are) a subset(s) of S?

A. $(-\frac{1}{2}, -\frac{1}{\sqrt{5}})$

B. $(-\frac{1}{\sqrt{5}}, 0)$

C. $(0, \frac{1}{\sqrt{5}})$

D. $(\frac{1}{\sqrt{5}}, \frac{1}{2})$

Mathematics Numerical (Maximum Marks: 24)

Question No. 1

Numerical Type

The answer has to be filled into the input box provided below.

On the ellipse $E = \frac{x^2}{64} + \frac{y^2}{9} = 1$, tangents drawn at the point $P_1, P_2, P_3, \ldots \ldots P_n$ on the ellipse intersecting the major axis at $T_1, T_2, T_3 \ldots \ldots, T_n$ respectively. If the value of $\sum_{i=1}^{n} \frac{\text{Area}(\Delta P_i T_i S) \cdot \text{Area}(\Delta P_i T_i S')}{(P_i T_i)^2} = 18$. where S and S' represents the foci of the ellipse, then ' n ' equal to:

Question No. 2

Numerical Type

The answer has to be filled into the input box provided below.

Let $S_1 \equiv x^2 + y^2 - 4x - 8y + 4 = 0$ and S_2 its image in the line $y = x$. The radius of the circle touching $y = x$ at $(1, 1)$ and orthogonal to S_2 is $\frac{3}{\sqrt{\lambda}}$, then $\frac{\lambda^3}{5}$ is equal to

Question No. 3

Numerical Type

The answer has to be filled into the input box provided below.

If z is a complex number and the minimum value of $|z| + |z - 1| + |2z - 3|$ is λ and if $y = 2[x] + 3 = 3[x - \lambda]$, then find the value of $\frac{[x+y]}{10}$ (where [.] denotes the greatest integer function)

Question No. 4

Numerical Type

The answer has to be filled into the input box provided below.

If $\begin{bmatrix} 1 & 2 & a \\ 0 & 1 & 4 \\ 0 & 0 & 1 \end{bmatrix}^n = \begin{bmatrix} 1 & 18 & 2007 \\ 0 & 1 & 36 \\ 0 & 0 & 1 \end{bmatrix}$ then find the value of $\frac{(n+a)}{100}$.

Question No. 5

Numerical Type

The answer has to be filled into the input box provided below.

Let $A = \begin{bmatrix} \sec(x) & \cos(x) & \sec^2(x) + \cot(x) \cdot \operatorname{cosec}(x) \\ \cos^2(x) & \cos^2(x) & \operatorname{cosec}^2(x) \\ 1 & \cos^2(x) & \cos^2(x) \end{bmatrix}$

Where $x \in (0, \frac{\pi}{2})$, then the value of $\left| AA^{\top}(A^{-1}) \right|^2$ is (where $|P|$ is determinant value of matrix P)

Question No. 6

Numerical Type

The answer has to be filled into the input box provided below.

Let $k_1, k_2 \in R$ be such that $\lim_{x \to 0} \frac{x^2 \tan(k_2 x)}{k_1 x - \tan x} = 1$. Then $[k_1 + k_2]$ equals (where [.] represents greatest integer function)